Pergamon

0301-9322(93)E0014-V

Int. J. Multiphase Flow Vol. 20, No. 1, pp. 217–218, 1994

0301-9322/94 $6.00 + 0.00

BOOK REVIEW

Macrotransport Processes, by H. BRENNER & D. A. EDWARDS. Butterworth/Heinemann (1993).

My first encounter with the content of this book was the paper by Brenner and Gaydos, which was published in 1977 in the *Journal of Colloid and Interface Science*. I must admit that I was thoroughly impressed by the elegant simplicity of the analysis, which took into account the complexity of the real situation, including all the hydrodynamic interactions between the particle and the wall.

I received similar impressions on two additional occasions. The now classical paper on Taylor dispersion in porous media, which was published in the *Philosophical Transactions of the Royal Society*, was memorable for the totally new viewpoint which was developed on porous media and the fact that the terrifying complexity of this problem could be grasped by such a powerful analysis. My last strong impression was created by the papers published through 1986–1989 on the properties of macromolecules: though these papers still await practical implementation, I am certain that some fascinating theoretical results will be derived with this method in the future with interesting application to polymer theory.

These different phenomena can actually be addressed by the same mathematical tool, namely the method of moments. Let us assume that the position of a particle in space is characterized by the two variables $\mathbf{Q}$ and $\mathbf{q}$. The global variable $\mathbf{Q}$ is generally unbounded, while the local variable $\mathbf{q}$ is generally bounded. This particle, which undergoes Brownian motion, is introduced at point ($\mathbf{Q}_0$, $\mathbf{q}_0$) at time t_0. In order to calculate the major features of the probability distribution $P(\mathbf{Q}, \mathbf{q}, t \mid \mathbf{Q}_0, \mathbf{q}_0, t_0)$ defined at point ($\mathbf{Q}$, $\mathbf{q}$) and time t, one calculates its successive moments $\mathbf{M}_m$ $(t \mid \mathbf{Q}_0, \mathbf{q}_0, t_0)$:

$$\mathbf{M}_m = \int_{\mathbf{Q}} \int_{\mathbf{q}} \mathbf{Q}^m P \, \mathrm{d}\mathbf{q} \mathrm{d}\mathbf{Q}.$$

Although it is more natural, as written above, to start the integration by the local variables, the trick is extremely simple and amounts to inversion of the two integrals by introducing the local moments $\mathbf{P}_m(\mathbf{q}, t \mid \mathbf{Q}_0, \mathbf{q}_0, t_0)$:

$$\mathbf{P}_m(\mathbf{q}, t \mid \mathbf{Q}_0, \mathbf{q}_0, t_0) = \int_{\mathbf{Q}} \mathbf{Q}^m P \, \mathrm{d}\mathbf{Q}.$$

It can easily be shown that the local moments obey a partial differential equation of the convection–diffusion type. This equation can be derived from the convection–diffusion equation which governs the concentration $P(\mathbf{Q}, \mathbf{q}, t \mid \mathbf{Q}_0, \mathbf{q}_0, t_0)$. The global moments are then calculated by an integration over the local variable $\mathbf{q}$.

This simple inversion of the integrals proves to be extremely fruitful and it can be applied to a large variety of physical situations. Of course, the integration over the local scale provides the macroscopic quantities of interest, such as the dispersion tensor. The exploration of all the different situations which can be addressed by this method constitutes the major portion of the material in this book—which is reviewed briefly below.

This book consists of three major parts. The first, which is also the largest, addresses the dispersion of real material such as a Brownian molecule; while the second is more abstract, in the sense that immaterial quantities, such as energy and momentum, are the dispersing quantities. The last part is fundamental; the relationships between different approaches of Brownian motion are analysed from a theoretical viewpoint. Two appendices complete this important volume which contains more than 700 pages. It will become obvious to readers that the book is written in a very pedagogical manner; but the price one pays for clarity is a certain redundancy between the various chapters. By the same token, these chapters can be read independently by an experienced reader who is looking for particular information on a given topic.

Part 1 can be roughly subdivided into the following subparts. Chapters 1 and 2 are introductory: Chapter 1 gives some interesting details about the historical developments of the theory, as well as some of its major practical applications; and Chapter 2 analyses the simplest possible example of unidirectional dispersion in a straight tube. A useful distinction is made between dispersion in continuous media, such as dispersion of a Brownian particle in an infinite fluid, and dispersion in discontinuous media, the prototype of such media being spatially periodic porous media. The general organization of these chapters is roughly the same. After a general exposition of the problem, the method of moments is applied in the long time limit. The equations which govern the local moments are obtained, as well as the macroscopic quantities of interest—such as the mean solute velocity and dispersivity; then the macrotransport equation, which governs the transport at the macroscale, is derived. The chapter and its major results are summarized. Examples are given to illustrate the methodology. The rest of Part 1 is devoted to more complex physical situations. The authors show how these complications can be handled within the same framework. This includes surface and interfacial transport, time-periodic processes, dispersion of fluctuating Brownian–particle clusters and chemically reactive systems. This is an impressive list of the potential applications of this approach.

Part 2 is more abstract, in the same sense that it provides essentially a novel way to derive known results. The general approach can be summarized roughly as follows. Conservation equations coupled with linear constitutive laws (such as Fourier's law for heat and the linear stress–strain relationship for Newtonian fluids) inevitably contain a Laplacian operator which can be interpreted in terms of the diffusion of a particular quantity. This approach is applied to energy and momentum in Chapters 10 and 11, respectively.

Part 3 is still more abstract and addresses the technical problems of the equivalence between various methods of analysing random processes at various time scales. Three time scales are usually distinguished, which correspond to the temporal domains of validity of the Langevin, Fokker–Planck and Smoluchowski equations.

This brief overview does not do complete justice to the rich content of this book, which give rise to an impressive number of applications. Some of them have already been done and one may regret not finding them in this book. This would have provided some facts and figures and it would have considerably enlarged the audience to include the practitioners. In the same vein, but with a different audience in mind, it is perhaps a pity that the competitive approaches (mainly homogenization) have not been compared to the method of moments. It could be quite interesting to show that the homogenization method based on a spatial analysis is mostly equivalent to the method of moments based on a time analysis.

The presentation of this book is quite pleasant and the number of typographical errors remarkably small. However, I was able to find an amusing error on p. 62, where the University of Brussels is located in Denmark! I did not know that the linguistic division went so far!

I would like to conclude this review by saying that the authors have succeeded in putting together a truly outstanding book, mostly based on their own research. It summarizes and puts in a unique framework a large number of papers which attracted a lot of attention from the scientific community. For these reasons, it will prove useful both to the expert, who will easily find in it what he is looking for, and to the beginner, who will have a unique reference to study.

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